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# Analysis of Variation of Stiffness Derivative With Mach Number and Angle of Attack for a Supersonic Flow

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***Abstract:*** *In this paper unified supersonic theory is used to derive the expressions of Stiffness derivative of a wedge in Hypersonic flow. This paper demonstrates the effects of pivot position on the stffness derivative with Mach number and incidence angle. From the above discussion it is seen that behaviour of the stiffness derivative is the same for h = 0 to 0.4 and also for h = 0.8, 0.9, and 1.0. Due to the pivot position variations there is a significant variation of Stiffness derivative with Mach number and incidence angle.*
***Keywords:*** *Incidence angle, Stiffness derivative, Supersonic flow, variation*

## I. Introduction

Ghosh and Mistry [2] have given a theory for the quasi steady flow over the oscillating 2-D wedge which has been further extended by Ghosh [3] to solve the axi-symmetric flow problem. Crasta and khan [4 to 28] have taken Ghosh's [1] large deflection similitude into consideration to develop an unsteady piston theory for the prediction of stability derivatives of oscillating planar and non-planar wedges. Here an attempt is made to analyze the stiffness derivative variation with Mach number and incidence angle for various pivot position from h = 0 to 1.

## II. Analysis

The exact expression for pressure ratio using the second order shock wave expansion theory, can be written in quadratic form in pressure ratio since the piston Mach number $M_p = u_p / a_\infty \gg 1$, yielding :

$$\frac{p}{p_\infty} = 1 + AM_\infty{}^2 sin^2\alpha + AM_\infty sin\alpha\sqrt{B + M_\infty{}^2 sin^2\alpha} \qquad (1)$$

A piston theory which has been used in equation (1) has been extended to supersonic flow. The expression is given below:

$$\frac{p}{p_\infty} = 1 + A\left(\frac{M_p}{\cos\phi}\right)^2 + A\left(\frac{M_p}{\cos\phi}\right)\left(B + \left(\frac{M_p}{\cos\phi}\right)^2\right)^{\frac{1}{2}} \qquad (2)$$

Where $p_\infty$ is free stream pressure, $A = \frac{(\gamma+1)}{4}, B = (4/(\gamma+1)^2, \gamma$ is the specific heat ratio and $M_p =$ the local piston Mach number normal to the wedge surface.

The nose down moment is given by:

$$-m = \int_0^L (x - x_\circ) p dx \qquad (3)$$

The stiffness derivatives is given by:

$$-C_{m_{\alpha\circ}} = \frac{1}{\frac{1}{2}\rho_\infty U_\infty{}^2 L^2}\left(-\frac{\partial m}{\partial \alpha_\circ}\right)$$

Evaluated at $\alpha = \alpha_0$ and q = 0, Piston Mach number

$$M_p = \frac{1}{a_\infty}[U_\infty Sin^2\alpha + (x - x_o)\, q] \qquad (4)$$

Defining $x_0 = hL\cos^2 \propto_0$, $c = L\cos \propto_0$, $S^1{}_1 = \frac{M_\infty \sin\alpha_O}{\cos\varphi}$, the stiffness derivative of a wedge becomes

$$-C_{m_\propto} = \left[\frac{(\gamma+1)}{M_\infty \cos\alpha_\circ \cos\emptyset}\right] F(S^1{}_1) \left(\frac{1}{2} - h\cos^2\alpha_\circ\right) \qquad (5)$$

Where

$$F(S^1{}_1) = 2S^1{}_1 + \frac{B + 2S^1{}_1{}^2}{\sqrt{B + S^1{}_1{}^2}}$$

Graphs have been plotted for Stiffness derivative versus Mach number and incidence angle for variant pivot positions and results have been discussed.

## III. Results And Discussion

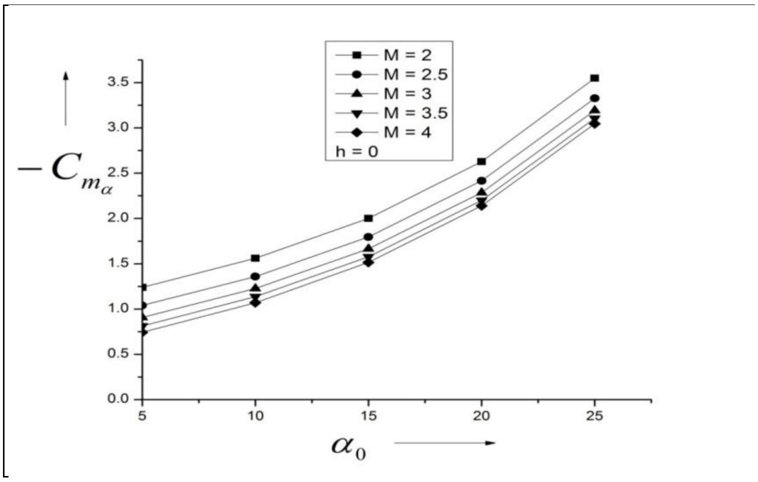


**Fig1:** Stiffness derivative versus incidence angle for pivot position h = 0

Examination of Fig.1 shows the enhancement of stiffness derivative with incidence angle. Futher there is decrement in Stiffness derivative with Mach number as the stiffness derivative is considered at the nose of the wedge i.e. at h = 0. The value of Stiffness derivative ranges between 1.25 to 0.75 for angle of attack of 5 degrees as Mach number increases from M = 2 to M = 4 there is continous decrement in the stiffness derivatives as the inertia terms as the Mach number is in the denominator of the expression. Also, due to the variation in the pressure distribution on the surface of the wedge.

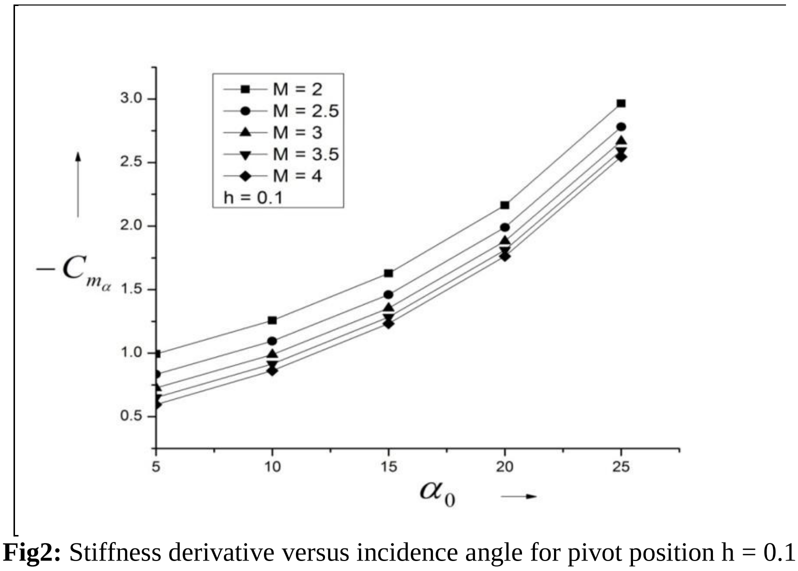


**Fig2:** Stiffness derivative versus incidence angle for pivot position h = 0.1

In Fig.2, the pivot position is shifted to h = 0.1. It is evident as the pivot position moves from the nose, the Stiffness derivative decreases futher with the increase in the Mach number and incidence angle. Even though the Stiffness derivative incrases with incidence angle and decrease with Mach number it is seen that the range of value is reduced when compared to that at the nose, this trend due to the shift in the plan form area of the wedge.

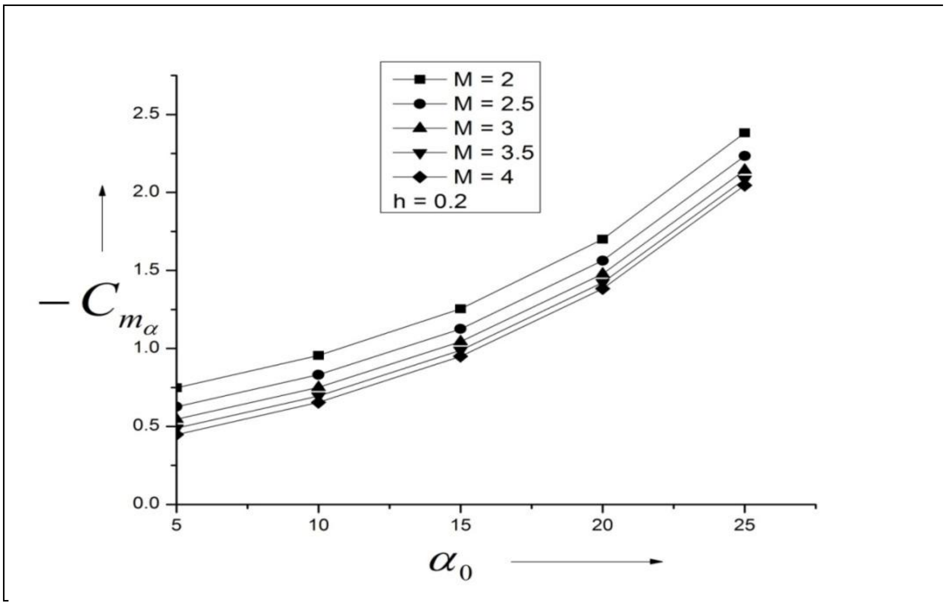


**Fig3:** Stiffness derivative versus incidence angle for pivot position h = 0.2

Fig.3 represents the Stiffness derivative variation with incidence angle for pivot position h = 0.2. From the figure it seen that there is significant decrease in the value of Stiffness derivative for incidence angle 5 degrees and Mach number M = 2 is visible and in the proportion the trend continues and the reasons for this trend holds as discuss earlier.

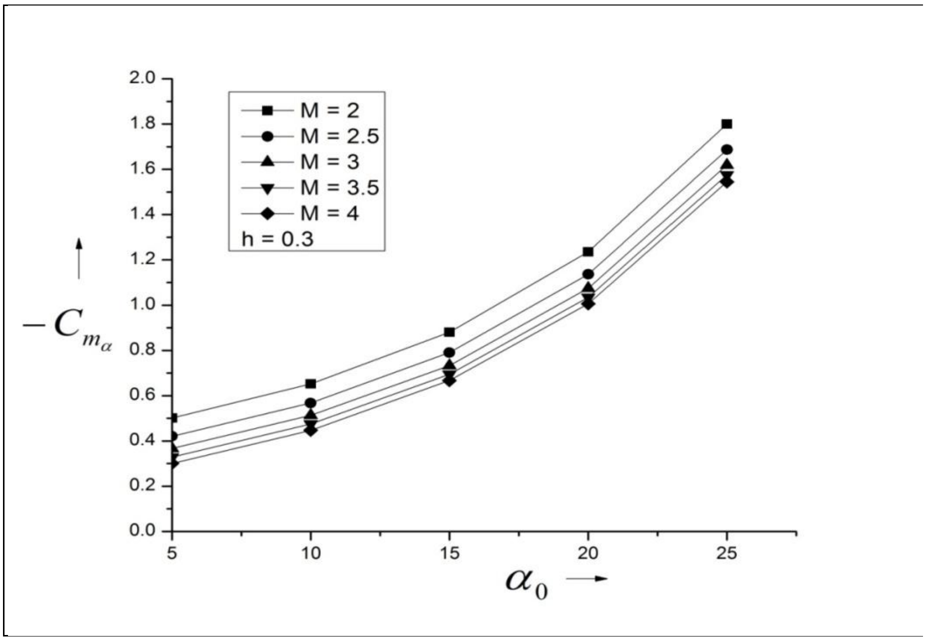


**Fig.4:** Stiffness derivative versus incidence angle for pivot position h = 0.3

Fig.4 shows the stiffness derivative variation with the incidence angle for pivot position h = 0.3. Here once again it is seen that there further decrement in the values of the stiffness derivatives due to the continous decrease of the plan form area.


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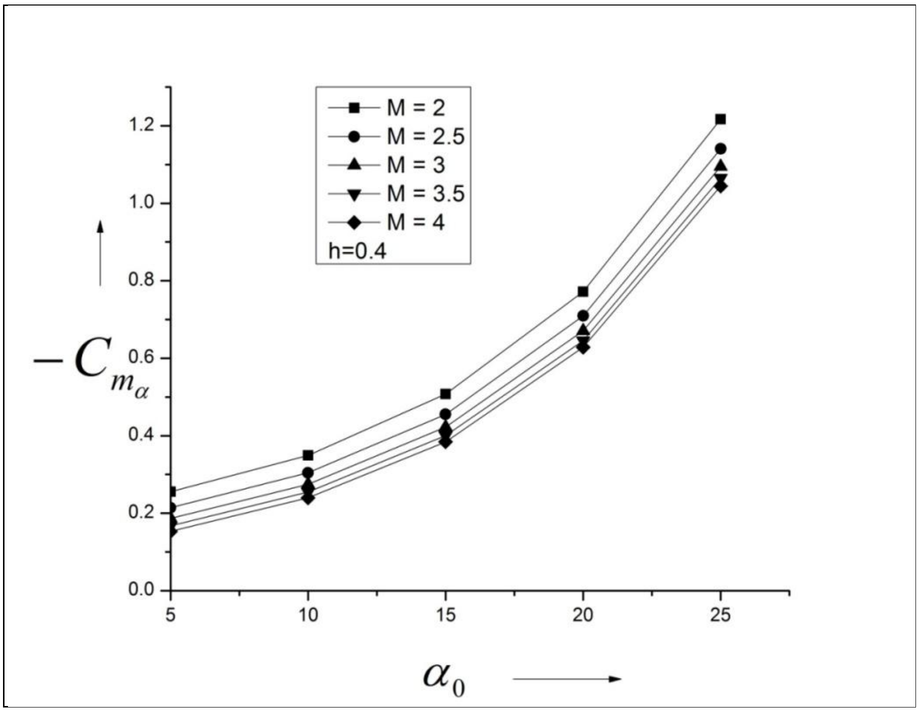


**Fig.5:** Stiffness derivative versus incidence angle for pivot position h = 0.4

Fig.5 presents the results of Stiffness derivative with incidence angle for pivot h = 0.4. At incidence angle 5 degrees there is further decrease in the value of Stiffness derivative when the pivot position moves from leading edge to the trailing edge and the same reason is valid as discussed earlier.

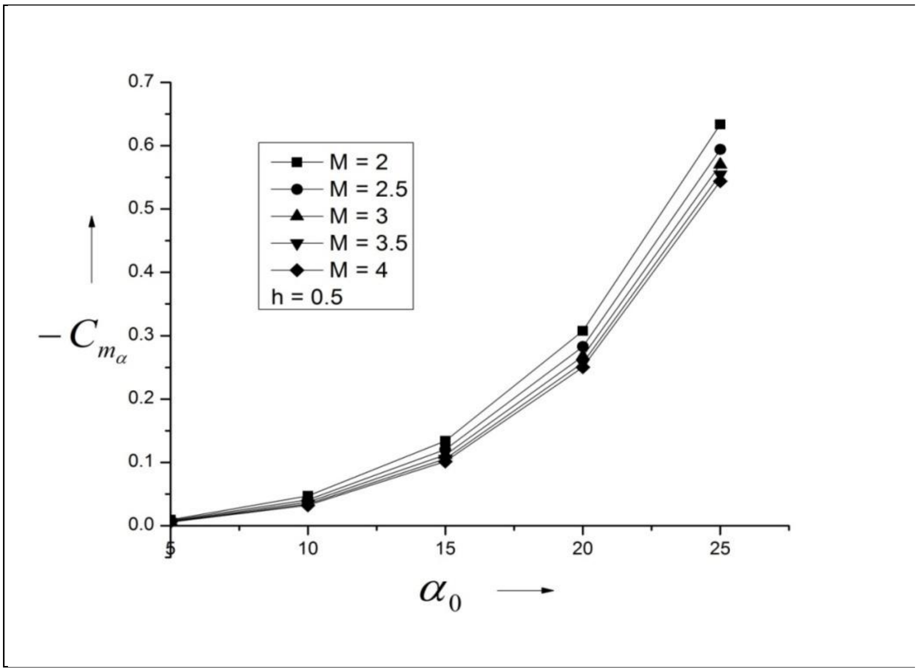


**Fig. 6:** Stiffness derivative versus incidence angle for pivot position h = 0.5

In Fig.6 it is clearly visible that there is no much difference in Stiffness derivative and incidence angle 5 to 10 degrees since the pivot position h = 0.5 is very close to the aerodynamic center of pressure. It is seen that as we are progressively moving from leading edge h = 0 towards trailing edge h = 1, there is a continuous decrease in the plan form area of the wedge resulting in change of surface pressure and hence the variations in the stiffness derivative.

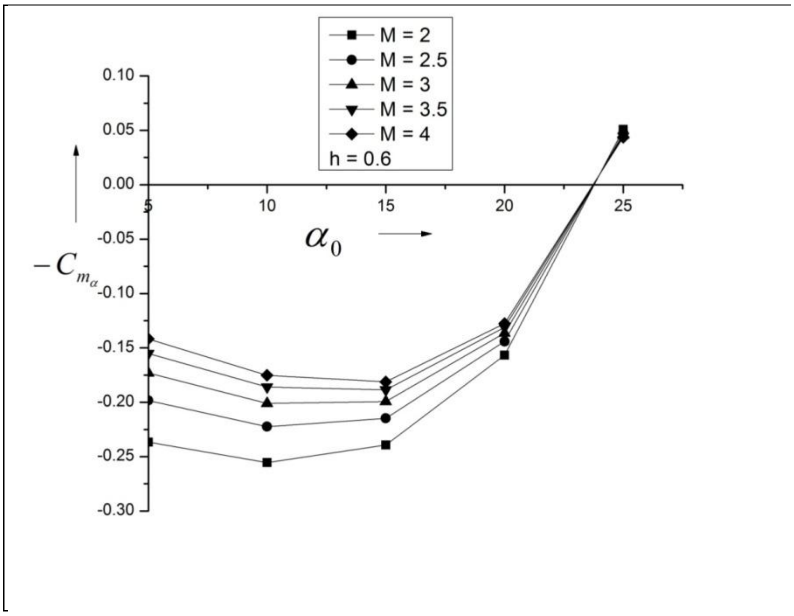


**Fig.7:** Stiffness derivative versus incidence angle for pivot position h=0.6

Fig.7 shows the Stiffness derivative variation with the incidence angle for pivot position h = 0.6. A different behavior is observed. This trend is due to location of pivot position which is beyond the center of pressure and more so due to the shift of the larger area towards the leading edge and the plan form area available to balance the negative static margin is not available, however at angle of incidence of 23 degrees and above there a shift in the center of pressure and hence in the sign of the stiffness derivative.

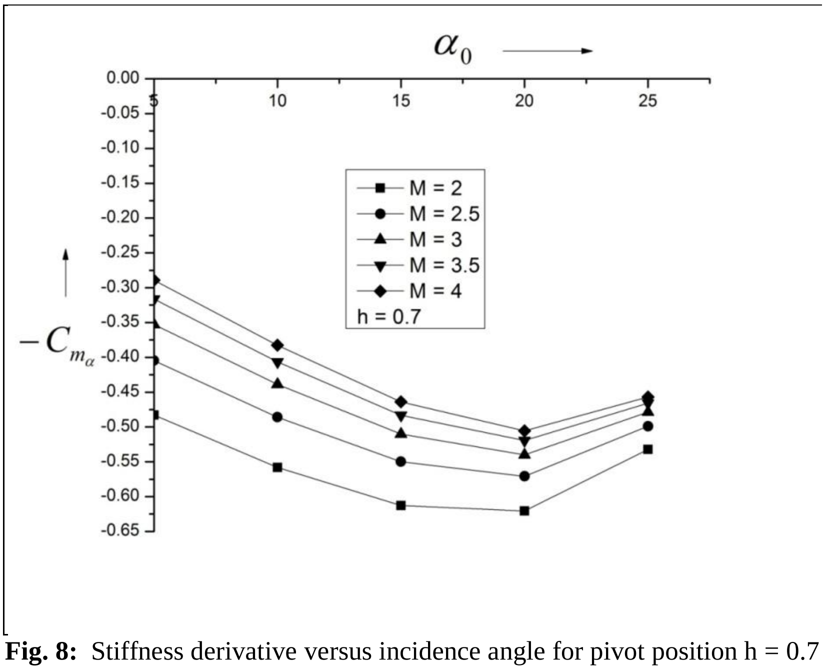


**Fig. 8:** Stiffness derivative versus incidence angle for pivot position h = 0.7

Fig.8 shows the Stiffness derivative variation with incidence angle for pivot position h = 0.7 from the nose. The value of stiffness derivative decreases as the Mach number increases and reduces with the angle of attack. This trend is due to the major shift in the pan form area, location of the pivot position which behind the center of pressure and under these circumstances when stiffness derivatives are evaluated they have large negative values as compared to the lower values of the pivot position and with increase in angle of attack flow field is such that even incidence angle is unable to give any relief. Normally angle of attack has been used to stabilize the flying objects. This is possible when we consider the body as whole and not a section of the moving vehicle.

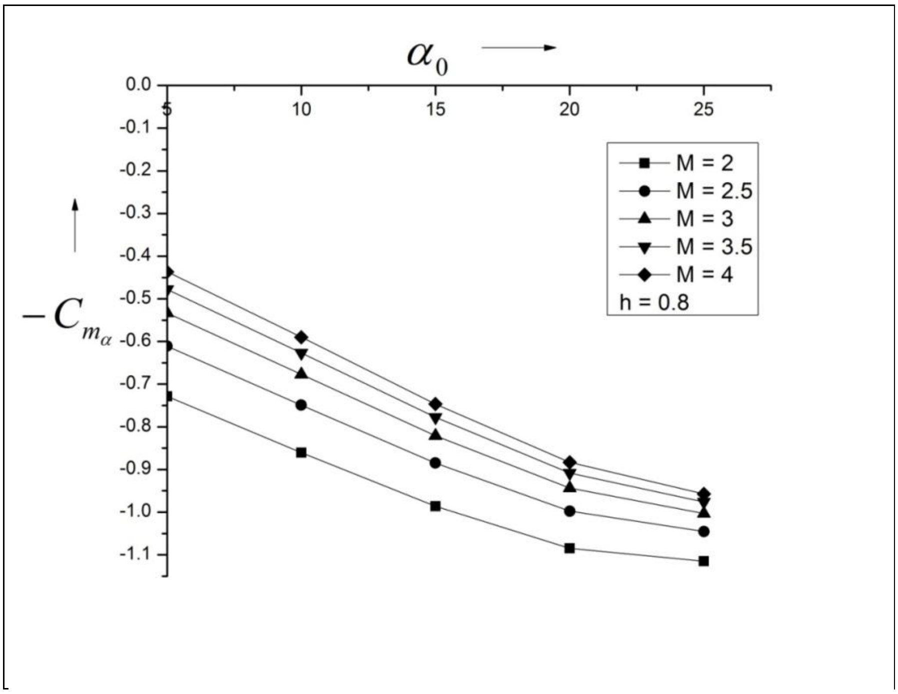


**Fig. 9:** Stiffness derivative versus incidence ange for pivot position h = 0.8

Fig.9 shows the stiffness derivative variation with angle of incidence for pivot h = 0.8. From the figure it is seen that there a linear decrement in stiffness derivative with respect to incidence angle. The range of values for stiffness derivative has further gone down negative, and reasons for this trend holds as discussed above.

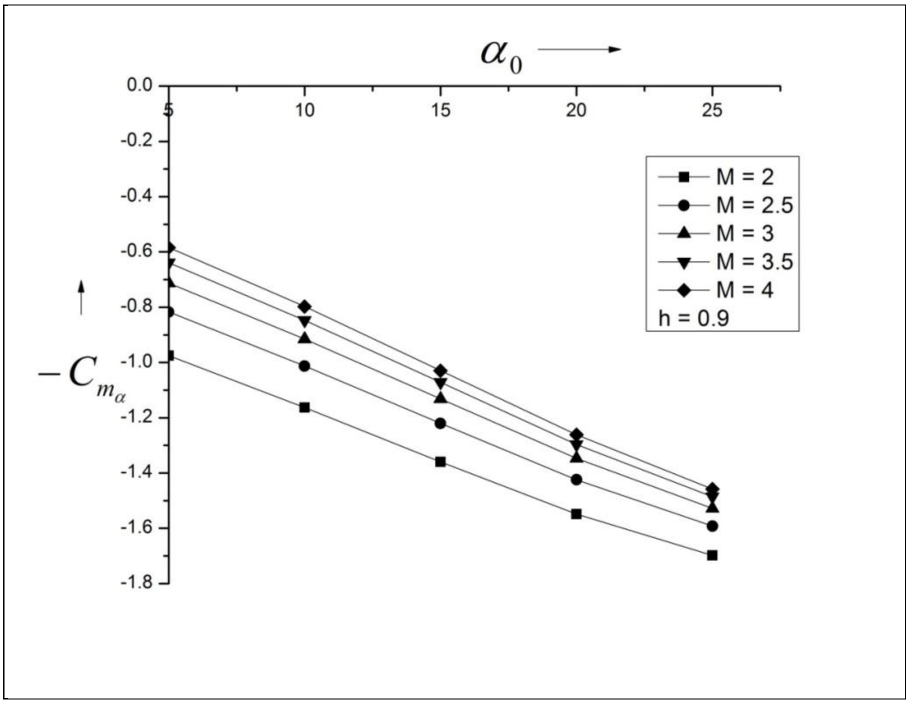


**Fig.10:** Stiffness derivative versus incidence angle for pivot position h = 0.9

Fig.10 shows the Stiffness derivative variation with respect to angle of attack for pivot position h = 0.9. It is observed that at the pivot position 90% away from the nose shows tendency of linear decrement in Stiffness derivative with increase in angle of incidence there is further decrement in the values, however, this decrease is linear.

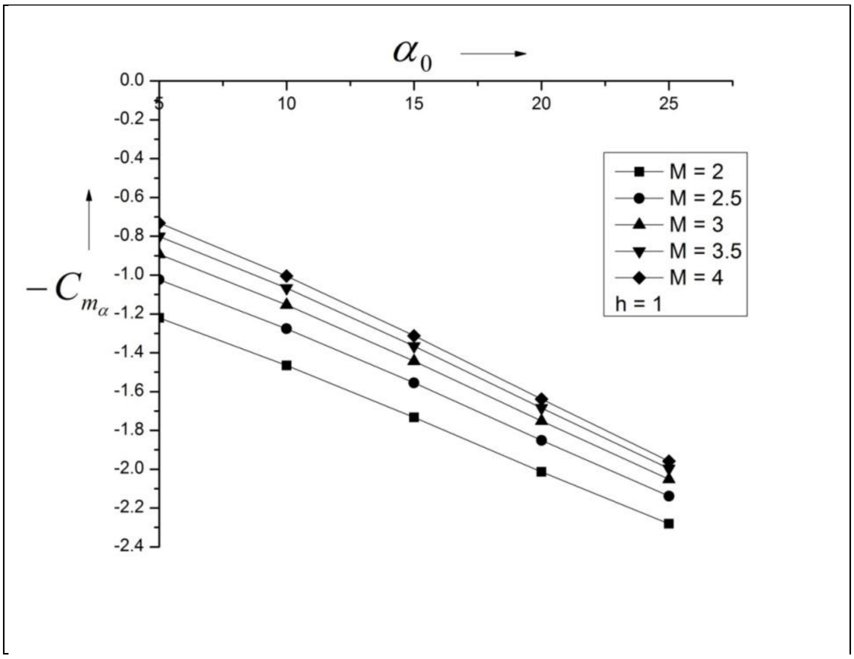


**Fig.11:** Stiffness derivative versus incidence angle for pivot position h = 1

Fig.11 shows the Stiffness derivative variation with angle of attack for pivot position h =1. The same trend is same as discused earlier with the exception that magnitude has changed due to change in the location of the pivot position and this pivot position is at the end of the wedge which is the trailing edge and no plan form area is available to counter the nose down moment.

## IV. Conclusion

- From the above discussion it is seen that behaviour of the stiffness derivative for h = 0, 0.1, 0.2, 0.3, and 0.4 is on the similar lines which are shown in figures from 1 to 5.
- For h = 0.5 the initial values are nearly zero, however due to the increase in the angle of attack there steep rise in the stiffness derivatives.
- For h = 0.6 the stiffness derivative has become positive becomes zero for angle of attack 24 degrees and then becomes negative at angle of attack beyond 24 degrees.
- For h = 0.7 there is linear decrement the stiffness derivative up to angle of attack 20 degrees and then starts increasing.
- For h = 0.8, 0.9, and 1.0 there is linear decrement in the stiffness derivative for all the three pivot positions, due to the change in the pivot position there is variation in the magnitude alone.

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